# Thermalization of heavy quarks in the QGP

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June 13, 2005

## 1  Introduction

Recent experimental results on the transverse-momentum spectra ($p_T$ spectra) and the elliptic flow, $v_2$, of non-photonic single electrons [K$^+$04, L$^+$04], whose main source at $p_T \gtrsim 2\text{GeV}/c$ is the semi-leptonic decay of open-charm (and bottom) mesons, indicate that $D$ mesons may have about the same $v_2$ as light mesons. Calculations within a quark-coalescence model [GKR04] show that this can be explained only under the assumption that charm quarks interact strongly within the QGP and are driven quickly towards thermal equilibrium (see Fig. 1).

Within transport models [Mol04, ZCK05] such a fast thermalization can only be achieved by large cross sections for charm-quark rescattering in the QGP. On the other hand, using a Langevin approach to describe such processes, in [MT04] it has been found that one needs to scale the pQCD-cross sections up by choosing unrealistically high coupling constants, $\alpha_s$, to obtain a large $v_2$ of charm quarks. In turn this leads to a small $R_{AA}$ for charm quarks at high $p_T$. Here, we assume the existence of $D$-meson like resonance states within the QGP as a microscopic explanation for such strong charm-quark rescattering effects in the hot and dense medium.

The idea of hadron-like resonances within the QGP is motivated by lattice-QCD studies of the corresponding spectral properties in the light-quark sector, which indicate that meson-like resonances "survive" within the quark-gluon plasma up to temperatures of about $2T_c$ [AH03, KL03]. This has been used to provide a nonperturbative mechanism for an enhancement of partonic cross sections within the QGP, leading to a rapid thermalization of the bulk matter at RHIC [SZ04, BLRS04, MR05] as required in hydrodynamic models.

Also the notion of charmonium resonances in the QGP [DKPW03, UNM02] has been applied earlier to assess $J/\psi$ production at SPS and RHIC. In these studies, the regeneration of charmonia by $\bar{c}c$-resonance formation within the QGP has been found to be the dominant contribution to the total yield of $J/\psi$'s in central collisions at RHIC [GRB04].

Here, we analogously assume the presence of $D$-meson like resonances within the QGP as a possible nonperturbative mechanism for rescattering of charm quarks with light (anti-) quarks in the QGP [HR05].
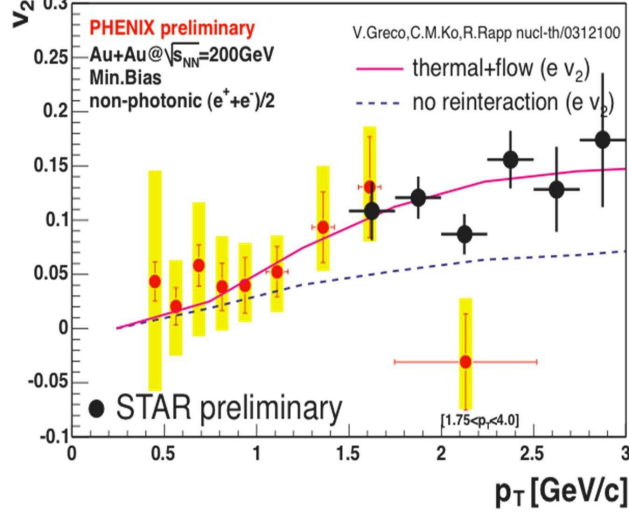
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Figure 1: The elliptic flow, $v_2$, of non-photonic electrons measured by the STAR and PHENIX collaborations at RHIC [K$^+$04, L$^+$04]. A calculation within a coalescence model [GKR04] shows that these data are only explainable if charm quarks are driven towards thermal equilibrium with the bulk partonic matter at RHIC.

## 2 Model for $D$-mesons within the QGP

To describe $D$-meson like resonances within the QGP we utilize a model, based on chiral SU(2)$_V$ $\times$ SU(2)$_A$ symmetry in the light-quark sector. The free chirally symmetric Lagrangian for pseudoscalar $D$-mesons, their scalar partners, the vector mesons $D^*$ and their axial-vector partners, and for the quarks is given by

$$\mathscr{L}_D^{(0)} = \bar{q}\mathrm{i}\partial\!\!\!/ q + \bar{c}(\mathrm{i}\partial\!\!\!/ - m_c)c + \sum_{i=1}^{2}[(\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i] + \text{massive (pseudo-)vectors } D^*. \quad (1)$$

Here, $q$ denotes the light-quark isospin doublet, $c$ the charm quark field, $\Phi_i$ two isospin-doublet fields, describing pseudoscalar anti-$D$ mesons and their chiral partners. In addition we introduced also massive vector- and pseudo-vector fields for the anti-$D^*$ mesons and their chiral partners.

The interaction Lagrangian is restricted by chiral symmetry and parity conservation. Further, to ensure transversality of the vector- and axial-vector mesons, we used the interactions from heavy-quark effective theory (HQET):

$$\begin{aligned} \mathscr{L}_{\text{int}} = &- G_S \left( \bar{q}\frac{1+\slashed{v}}{2}\Phi_1 c_v + \bar{q}\frac{1+\slashed{v}}{2}\mathrm{i}\gamma^5 \Phi_2 c_v + h.c. \right) \\ &- G_V \left( \bar{q}\frac{1+\slashed{v}}{2}\gamma^\mu \Phi_{1\mu}^* c_v + \bar{q}\frac{1+\slashed{v}}{2}\mathrm{i}\gamma^\mu\gamma^5 \Phi_{2\mu}^* c_v + h.c. \right). \end{aligned} \quad (2)$$

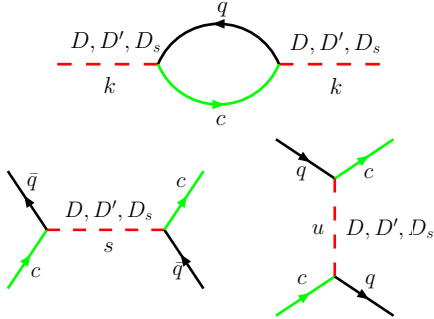Here, $v$ denotes the four velocity of the heavy quark. From spin symmetry of HQET, we also infer

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Figure 2: Top row: The self-energy diagram for the $D$-meson like resonances within the quark-gluon plasma. Bottom row: Charm-quark scattering off light (anti-) quarks through $D$-meson-like resonances. The $D$-meson propagator has been dressed with the one-loop self-energy diagram.
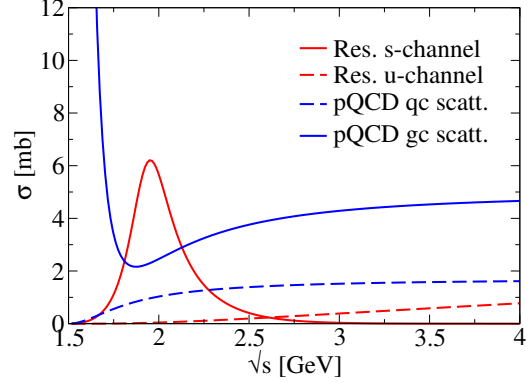


Figure 3: Cross sections for rescattering of charm quarks due to elastic perturbative-QCD processes and to resonance scattering.

$G_S = G_V$. This Lagrangian should be understood as a model for quasi particles within the quark-gluon plasma. Particularly chiral symmetry has to be realized in the unbroken (Wigner-Weyl) phase.

To account for the finite width of the $D$-meson like resonances, we calculated the one-loop self energy within this model (see upper diagram in Fig. 2). The width is given by the imaginary part of the self-energy, $\Gamma = -\operatorname{Im}\Pi/m_D$, which corresponds to the decay of the open-charm mesons to $c\bar{q}$ pairs, i.e., the cut of the self-energy diagram.

In Fig. 3 the cross sections for $c$-quark scattering with light quarks according to the resonance-scattering process within our $D$-meson model is compared with those from the corresponding leading-order perturbative-QCD contributions. The pQCD matrix elements were taken from [Com79] including a screening mass in leading hard-thermal-loop order, $\mu_{\mathrm{gluon}} = gT$, within the $t$-channel gluon-exchange diagrams.

## 3   Charm-quark rescattering in the QGP

To assess the reinteraction of charm quarks within the QGP, we use a Fokker-Planck equation, which is derived from the Boltzmann equation under the assumption that the scattering of the heavy quarks with the light constituents of the QGP is dominated by soft processes [Sve88]. Under the simplifying assumption of constant coefficients it reads

$$\frac{\partial f(t,\vec{p})}{\partial t} = \frac{\partial}{\partial p_i}\left[\gamma p_i + D\frac{\partial}{\partial p_i}\right]f(t,\vec{p}). \tag{3}$$
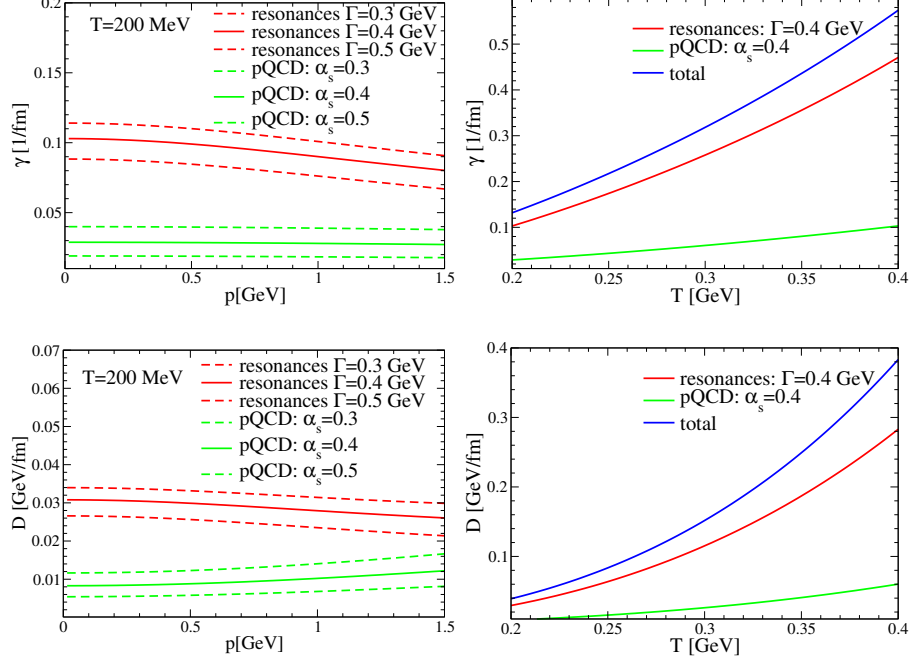
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Figure 4: The Fokker-Planck coefficients $\gamma$ (top row) and $D$ (bottom row) as function of momentum (left column) and temperature (right column).

The coefficients $\gamma$ and $D$ are directly related with the microscopic scattering processes of the charm quarks with the light (anti-) quarks within the QGP (see the two diagrams at the bottom of Fig. 2). The friction or drag coefficient, $\gamma$, describes the rate at which an initial momentum of a charm quark relaxes to the value corresponding to the velocity of the bulk medium. The momentum-diffusion coefficient, $D$, is a measure for the rate with which an initially sharp charm-quark-momentum distribution broadens to its equilibrium value. The equilibrium solution of the equation is a Maxwell-Boltzmann distribution with a temperature, given by Einstein's dissipation-fluctuation relation

$$T = \frac{D}{m\gamma}. \tag{4}$$

As can be seen in Fig. 4, the contributions from resonance scattering to the friction coefficient is by a factor of 2 to 3 higher than those from the perturbative-QCD processes. The Fokker-Planck coefficients are quite insensitive to variations of the coupling constants in both the resonance model and pQCD. Naively one would expect a variation with $G^4$ ($G$: $c\bar{q}D$-coupling constant) and $\alpha_s^2$ as the leading order of the respective squared matrix elements suggests, but this effect is compensated by the width of the resonances, which rises with $G^2$, and the increase of the gluon-screening mass with $\sqrt{\alpha_s}$. The resonance-scattering contribution to the Fokker-Planck coefficients is dominated by the $s$-channel diagram (see Fig. 2), i.e., elastic scattering of $c$ quarks with light antiquarks. As shown in Fig. 3, this is not so much due to the magnitude of the corresponding cross section but its isotropy which makes this process more efficient for the equilibration of charm quarks with the bulk matter,
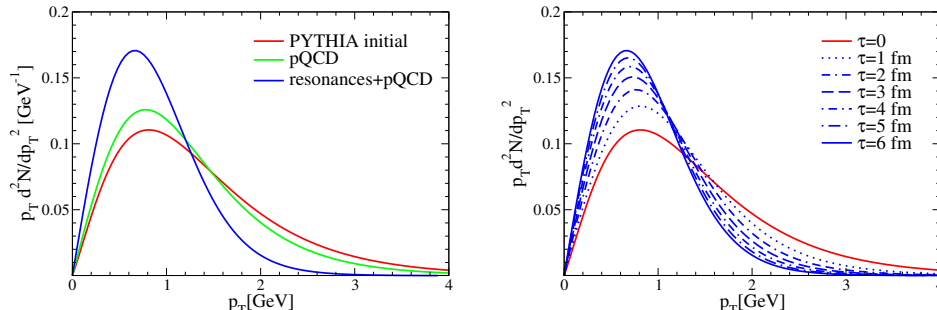
Figure 5: The time evolution of the charm-quark $p_T$ spectrum.

compared to the pQCD processes, whose cross sections are peaked in forward direction[1].

Since the Fokker-Planck coefficients are only weakly momentum dependent but change considerably with temperature, we solve the Fokker-Planck equation for momentum independent but time dependent coefficients [HR05]. Here, the time dependence enters the Fokker-Planck coefficients through that of the fireball temperature, which is determined from a simple fireball model [Rap01], parameterizing results of hydrodynamical calculations [KR03].

Assuming isentropic expansion, the temperature at each instant is then calculated from the total entropy of produced particles.

The initial QGP temperature is $T_0 \simeq 375$ MeV, decreasing to the critical temperature of $T_c \simeq 180$ MeV after about 3 fm/$c$, with further evolution in a hadron-QGP mixed phase for another 3 fm/$c$. The initial charm-quark distribution is taken from a PYTHIA calculation. As shown in the left panel of Fig. 5, using only the pQCD matrix elements for the evaluation of the Fokker-Planck coefficients, the final $p_T$ distribution has not changed significantly from the initial one. Taking into account the resonance-scattering contributions results in a distribution, which is much closer to an equilibrium Maxwell distribution. From its peak position one estimates an effective final temperature of about 290 MeV.

The right panel of Fig. 5 indicates that this equilibration process is most efficient in the early stages of the fireball evolution, as to be expected, because for these times the Fokker-Planck coefficients are large.

## 4    The flow of charm quarks

The results of the previous section cannot be compared to experimental data on $D$-meson flow, since (i) we have not taken into account the flow of the bulk matter, and (ii) the hadronization of charm quarks to $D$ mesons (and their subsequent decay to electrons) has to be evaluated.

To include radial and elliptic flow of the bulk medium, we generalize the fireball model. Now the fireball is modeled as an elliptical cylinder with an acceleration of the surface, which is adjusted to

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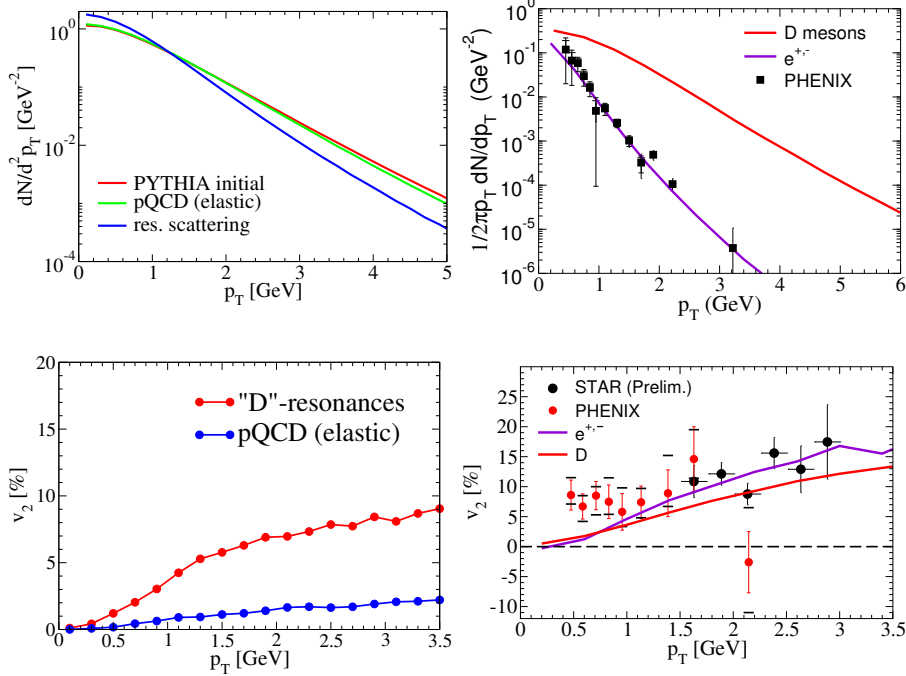[1]See also Che-Ming Ko's talk at this workshop.

Figure 6: Upper panel: Results for charm-quark (left) and $D$-meson and decay-electron-$p_T$ spectra (right). Lower panel: The elliptic flow, $v_2$ for charm quarks (left) and $D$-mesons and their decay electrons (right).

yield a final velocity corresponding to a minimum-bias $v_2$ of $\sim 5.5\%$ for the light quarks, estimated using a blast-wave model [HKH+01]. The velocity profile rises linearly with the radial distance from the center as inferred from hydrodynamical model calculations.

The Fokker-Planck equation is solved by simulating the analogous relativistic Langevin process. Since the Fokker-Planck coefficients are defined in the local rest frame of the heat bath, the Langevin process is first defined in this frame. The charm-quark momenta in the laboratory frame, where the bulk medium has the flow profile, given by the elliptic fireball model, are then obtained by the corresponding Lorentz boost [MT04].

The $p_T$ distribution and $v_2$ of charm quarks, resulting from this simulation were subsequently used in a coalescence model to obtain the properties of $D$ mesons and their decay electrons similar to [GKR04]. As shown in Fig. 6, the such obtained $p_T$ spectra and $v_2$ of electrons from $D$-meson decays are in agreement with data from STAR and PHENIX [K+04, L+04]. Note that within the coalescence model for hadronization the $v_2$ and $R_{AA}$ of $D$ mesons becomes higher than that of the charm quarks contrary to the behavior within fragmentation models.

# 5 Summary and Outlook

It was shown that the existence of $D$-meson like resonances within the quark-gluon plasma at temperatures of up to $2T_c$ leads to an explanation for the observed $v_2$ and $p_T$ spectra for single electrons at RHIC.

The reduction of equilibration time scales for charm quarks by a factor $\sim 3$ compared to those inferred from the pQCD processes is not so much due to a large magnitude of the resonance-scattering cross sections but their isotropy. With this equilibration mechanism, employing a fireball model which reflects the properties of hot and dense partonic matter under RHIC conditions, we find charm-quark properties ($p_T$ spectra and $v_2$), which are consistent with the measured $v_2$ and $p_T$ spectra of single electrons.

To assess the electron spectra and $v_2$ at higher $p_T$, also an analogous mechanism for bottom-quarks and $B$-meson like resonances has to be evaluated. We do not expect considerable rescattering effects for bottom quarks under RHIC conditions (for details see [HR05]).

At higher initial temperatures, as to be expected at the LHC, the existence of hadronic resonances within the QGP is unlikely, and one has to account for their disappearance in the early stages of the collision and their appearance at lower temperatures at later times of the fireball evolution.

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